Bayesian Network Model for Reliability Assessment of Power Systems

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Abstract — This paper presents an application of Bayesian networks (BN) to the problem of reliability assessment of power systems. Bayesian networks provide a flexible means of representing and reasoning with probabilistic information. Uncertainty and dependencies are easily incorporated in the analysis. Efficient probabilistic inference algorithms in Bayesian networks permit not only computation of the loss of load probability but also answering various probabilistic queries about the system. The advantages of BN models for power system reliability evaluation are demonstrated through examples. Results of a reliability case study of a multi-area test system are also reported.

Keywords — Power system reliability, LOLP, load uncertainty, area load dependency, Bayesian network.

I. INTRODUCTION

Probabilistic reliability indices serve as an accurate and consistent basis for assessing and comparing reliability of power systems, where component's outage and load demand are of stochastic nature. Analytical mathematical models [1-5] or Monte-Carlo simulation [6-7] are usually used for computing these indices. While existing methods can efficiently evaluate the probabilistic reliability indices of a power system, they usually reveal few details about the role of various components and subsystems in overall system reliability. Conversely, when the states of certain components in the system are known, existing methods do not offer a direct way to assess the conditional probabilities of the causes and/or the effects in the rest of the system. These conditional probabilities, if readily available, can be very useful for improving the assessment of system reliability. For example, one can use this information to determine the system weak points from the point of view of system reliability.

The proposed model for reliability assessment of power systems can address the issues mentioned above. This model is based on Bayesian network (BN) technique. It provides a probabilistic representation of the balance between the supply availability and the load demand at various points in the system. Probability of the loss of load states as well as other probabilities are computed efficiently by BN propagation algorithms [8,9]. The main advantages of the proposed BN model for reliability assessment of power systems can be summarized as follows:

• Simple and intuitive model building that is closely based on the physical power network topology.
• Easy incorporation of uncertainty and dependency in reliability assessment.
• Capability to monitor the probability of any variable in the system.
• Propagation of probabilistic information that allows a wide range of what-if analysis.

The proposed BN models can provide answers to important questions concerning power system reliability, such as:

• Is the given supply configuration reliable enough to meet certain demand forecast in the system?
• What is the most likely cause of some contingency condition at a certain site?
• Given some probability distribution of failure of a certain part of the network, what is the reliability of power supply to downstream sites?

II. BAYESIAN NETWORK MODEL OF POWER SYSTEM

A. Bayesian Networks

A Bayesian network is a directed acyclic graph in which nodes represent random variables and links represent direct probabilistic influences. The variables depicted in the BN represent key parameters characterizing the system being modeled. The direction of a link between two nodes is usually chosen to indicate a causal-effect or class-property relationship between variables denoted by these nodes. In a
BN model the conditional probability distribution \( P(B|A_1, \ldots, A_n) \) is used to quantify the strength of the influence of variables \( A_i \) on the variable \( B \). Nodes \( A_i \) are called the parents of \( B \) and \( B \) is called a child of each \( A_i \). A BN represents a complete probabilistic model of the system because the joint probability distribution of any elementary system state can be derived using the local conditional probability distributions and network topology [10]. This property suggests the use of BN models for computing the probability index for reliability assessment of power systems.

The BN representation of a probabilistic domain also greatly facilitates the updating of probabilities in response to some new knowledge, e.g. some observations, about the domain. The updated, or posterior, probabilities are computed by propagating the new knowledge throughout the network, observing during this process the local conditional probabilistic relations between variables [10]. Based on the posterior probabilities, one can reason about the most likely causes and/or effects of the observed facts. This feature makes BN modeling a powerful tool for diagnostic and predictive analysis.

Efficient procedures for performing probabilistic propagation have been developed [8,9]. Software packages for building Bayesian networks and performing computations are available for various computer platforms and can be readily utilized in a wide range of applications [11].

**B. Representation of loss of load probability in BN model**

In general form, probabilistic reliability indices of power system are computed as

\[
\text{index} = \sum_{i=1}^{n} P(S_i) \cdot F(S_i) \tag{1}
\]

where

- \( S_i \) - a vector of state variables characterizing the \( i \)th system state \((i = 1, \ldots, n)\)
- \( P(S_i) \) - probability of system state \( S_i \)
- \( F(S_i) \) - some performance function characterizing the reliability of the system.

Loss of load probability (LOLP) is defined as the probability of failing to meet the demand during the given period of time. If the loss of load probability LOLP is to be computed, the performance function can be chosen as a conditional probability:

\[
F(S_i) = P(LOL|S_i) \tag{2}
\]

where variable \( LOL = 1 \) if loss of load takes place in the system and \( LOL = 0 \) otherwise. Substituting (2) in (1) yields

\[
\text{LOLP} = \sum_{i=1}^{n} P(S_i) \cdot P(LOL|S_i) = P(LOL) \tag{3}
\]

So system LOLP is computed as the probability associated with the BN node representing variable \( LOL \).

**C. Basic building blocks of BN model for computing LOLP**

BN model for computing LOLP is build by connecting together sub-networks representing key components of the system based on the topology of physical power network. These basic building blocks includes capacity model, load model, convolution model and tie-line connection models.

**Capacity model**

The capacity model computes the probabilities of having various amounts of generating capacity available in the system. This is a discrete probabilistic distribution which depends on the forced outage rate (FOR) of generating units. These probabilities are usually evaluated using the well known recursive algorithm [13]. Identical capacity availability probabilities can be obtained from the BN capacity model shown in Figure 1. In this network, each variable \( U_i \) represents the capacity that the \( i \)th generating unit is ready to provide. \( U_i \) will equal the unit’s rated capacity if it is fully operational equal zero if it is on outage. The unit’s de-rated states can also be considered if necessary. The total available capacity of the whole system is represented in the network by variable \( G \)

\[
G = \sum_{i=1}^{n} U_i \tag{4}
\]

where \( n \) is total number of generating units in the system.

To compute the probability distribution of the total capacity \( G \) a BN with nodes for the \( U_i \)’s as direct parents of the node for \( G \) would be sufficient. However the size of the conditional probability matrix quantifying such a model becomes very large for networks with more than a few \( U_i \) nodes since this matrix is composed of conditional probability of \( G \) given every possible combination of states of the \( U_i \)’s. The use of intermediate variables \( I_j \)

\[
I_j = \sum_{i=1}^{n} U_i \tag{5}
\]

effectively reduces the overall size of the capacity model because every node in the network now has no more than 2 parent nodes. At the same time the resulting probability distribution of \( G \) remains correct. Such a model is called a causal independence model [12].

![Figure 1. Capacity model](image)

It’s easy to see that the state space of \( I_j \) and \( G \) is potentially very large for practical systems with hundreds of generating units. Truncation and aggregation can be used to keep the model within manageable size limits. The system states with the cumulative probability of occurrence less than...
a specified amount, e.g. $10^4$, are omitted. The remaining capacity state space is broken down into equally spaced intervals, e.g. 20 MW.

Load model

The load model computes the probability of having a certain load during specified time period. The probability distribution of load demand derived from the load duration curve (LDC) can be used directly in the BN. If a Gaussian distribution is assumed in modeling load uncertainty [5] the mean and the standard deviation (STD) are included in the BN model of load shown in Figure 2 where they serve as defining variables of the demand D. The conditional probability distribution assigned to D is chosen in such a way that for each load forecast mean and standard error, D will have a Gaussian probability distribution.

In a multi-area network, the correlation between load demand at different sites can affect the overall reliability index of the system [6]. This probabilistic correlation can be modeled by introducing a control variable in the BN model. The probability distribution of area demands can be made conditional on the value of the control variable. By assigning the various conditional probability distributions to this network, various degree of correlation between area load can be obtained.

![Figure 2. Load model](Image)

Interconnection model

In an interconnected system, surplus capacity at one site can be used to meet the load in some neighboring sites to improve the supply reliability. For a multi-area system, the availability of the assistance which neighboring areas can offer each other is determined by: i) the tie-line capacity; ii) the surplus capacity in the assisting system; and iii) the contractual agreement between utilities in different areas. For example, consider a system with two connected areas, 1 and 2. Assume that area 2 is selected to be the assisting site. Then the available network assistance in area 1 can be found as:

$$N_1 = \min \{B_2, L_{21}\}$$  \hspace{1cm} (6)

with

$$B_2 = \begin{cases} 0 & \text{if } G_2 < D_2 \\ G_2 - D_2 & \text{if } G_2 \geq D_2 \end{cases}$$  \hspace{1cm} (7)

where:

- $B_2$ - Capacity balance at assisting site (site 2)
- $L_{21}$ - Available transmission capacity between the areas
- $G_2$ - Available generation capacity at site 2.
- $D_2$ - Demand at site 2

The probability distribution of network assistance $N_i$ in this example can be obtained from the BN model depicted in Figure 3. Equations (6) and (7) define the relationships between variables in this network. This simple network can be modified to model more realistic multi-area connections, including two-way assistance between areas, capacity wheeling and more complex contractual agreements.

![Figure 3. Tie-line connection model](Image)

Convolution model

The Bayesian network shown in Figure 4 is equivalent to computing the system loss of load probability by convolution of the demand and available supply probability distributions. In this network, the system state variable LOL can take on one of the two values 0 and 1 which correspond to no loss of load and loss of load, respectively. One of the parents of LOL represents the local capacity balance $B$ which is computed as the difference between the local available capacity and the load demand. The other parent of LOL represents the capacity available from the network over transmission line $N$. Since a system is considered to be in the loss of load state whenever the demand exceeds the available supply, the conditional probability distribution $P(LOL|N,B)$ is determined as:

$$P(LOL)=\begin{cases} 1 & \text{if } G+N < D \\ 0 & \text{otherwise} \end{cases}$$  \hspace{1cm} (8)

For example, given $B$ and $N$ such that $B+N < 0$ the corresponding probabilities of LOL will be: $P(LOL=1|N,B) = 1$ and $P(LOL=0|N,B) = 0$. The conditional probability distribution $P(LOL|N,B)$ together with known probability distributions of $B$ and $N$ obtained from capacity model, load model and connection model are sufficient for computation of the probability of LOL. By definition, the LOLP index of the system will be identified as the probability $P(LOL=1)$.

![Figure 4. Convolution model](Image)

D. Validation of BN model with IEEE-RTS system

Validation of the BN methodology has been carried out by computing the loss of load expectation (LOLE) for the IEEE-RTS system [14] using the generation model, the load model and the convolution model. For both the daily and hourly load models, the results in the Table 1 show good agreement between indicators computed by BN model and the reference values for IEEE-RTS system.
IEEE-RTS with rounding [14] 1.43919 N/A
BN model with rounding step 20 MW 1.44918 9.19219
BN model with rounding step 50 MW 1.51456 9.74400

Table 1. LOLE result for IEEE-RTS system

<table>
<thead>
<tr>
<th>Parameter</th>
<th>LOLE, day/yr</th>
<th>LOLE, hr/yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>IEEE-RTS exact result [14]</td>
<td>1.36886</td>
<td>9.39418</td>
</tr>
<tr>
<td>IEEE-RTS with rounding [14]</td>
<td>1.43919</td>
<td>N/A</td>
</tr>
<tr>
<td>BN model with rounding step 20 MW</td>
<td>1.44918</td>
<td>9.19219</td>
</tr>
<tr>
<td>BN model with rounding step 50 MW</td>
<td>1.51456</td>
<td>9.74400</td>
</tr>
</tbody>
</table>

Table 2. System data of the sample system

<table>
<thead>
<tr>
<th>Area</th>
<th>Number of units</th>
<th>Unit cap. (MW)</th>
<th>FOR</th>
<th>Installed cap. (MW)</th>
<th>Daily peak load (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>10</td>
<td>0.02</td>
<td>75</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>25</td>
<td>0.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>10</td>
<td>0.02</td>
<td>60</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>20</td>
<td>0.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>20</td>
<td>0.02</td>
<td>130</td>
<td>90</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>30</td>
<td>0.02</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

E. A sample BN model of 3-area power system

The proposed model will be illustrated in details using an example of reliability analysis of a 3-area power system adapted from [13]. System data of the sample system are listed in Table 2. Tie-line capacities are depicted in Figure 5. Tie-lines FOR are assumed to be 0.001. Following additional assumptions on the inter-area assistance are made:

i. The surplus capacity in any area is available to assist neighbors. The amount of surplus is assumed to be split evenly between the assisted sites although other ratios can be used with no major modification to the model. we adopted this simple assisting power allocation rule to show the feasibility of the use of BN models in analysis of power system reliability. More elaborate treatment of inter-area assistance modeling can be found in [2].

ii. Area 1 provides some wheeling service so that area 3 can assist area 2 over tie the lines L3-1 and L1-2 in addition to the direct line L1-2.

Table 2. System data of the sample system

Area LOLPs are computed and analyzed using a BN model shown in Figure 6. The area convolution models are shaded in rectangles and the interconnection models are shaded in ovals. The transmission network topology allows to identify the source of available capacity. If a site is connected to other sites by multiple lines, the BN model will include a corresponding number of inter-connection model blocks similar to that in Figure 3. The BN model illustrated by this multi-area can be extended to make it applicable to system having other configurations.

The network in Figure 6 includes variables for modeling of the inter-area assistance and power wheeling. These variables are explained below:

\[ A_i \] – Available capacity in area \( i \) for assisting area \( j \).

\[ T_i \] – Assistance capacity from area \( i \) which is available at site \( j \). This variable is computed as:

\[ T_i = \min \{ A_i, L_i \} \tag{9} \]

\[ W_{ij} \] – Amount of assistance from area \( i \) for area \( k \) which is wheeled through area \( j \).

\[ X_i \] – Intermediate variable.

\[ X_i = \begin{cases} T_i & \text{if } T_i > T_i \\ -T_i & \text{otherwise} \end{cases} \tag{10} \]

Hence the network assistance received by areas \( i \) and \( j \) are computed as:

\[ N_i = \begin{cases} X_i & \text{if } X_i > 0 \\ 0 & \text{otherwise} \end{cases} \tag{11} \]

\[ N_j = \begin{cases} -X_i & \text{if } X_i < 0 \\ 0 & \text{otherwise} \end{cases} \tag{12} \]

The capacity availability probabilities for the areas and the area LOLP indices computed by the BN model are found to be identical to analytically computed values in [13]. The proposed BN model is especially suitable for computing LOLP under various scenarios of system load demand and generating unit availability because these parameters can be easily entered as input to the BN.
The proposed BN model can be used for other useful analysis, such as what-if contingency study. For example, to study the loss of load condition in area 1, variable LOL, in the BN in Figure 6 is set to 1 (true) and all the relevant probabilities are updated (Table 3). Results in Table 3 indicate that the failure of the 25MW unit in area 1 has the highest probability of occurrence given the loss of load condition in area 1. The prior failure probability of this generating unit was 0.02 while the calculated posterior probability increased to 0.99. The probabilities of failure at the 10MW units UI-10a, ..., UI-10e are also high. The individual impact of each failure state on the area LOLP can be tested easily on the BN model. The computation of LOLP in area 1 given the outage state of individual units shows that this index decreases significantly only in the case of simultaneous outage of UI-25 and one or more 10MW unit or the tie-lines. This result suggests that even though UI-25 is a weak point in this system its failure will not increase significantly the probability of the loss of load in the area 1 because of the presence of the inter-area assistance. This example shows that posterior probability analysis is an effective procedure for finding out the elements of the system that are most pertinent to the contingency being studied.

Table 3. Calculated probabilities of key variables in area 1

<table>
<thead>
<tr>
<th>Probability</th>
<th>Prior</th>
<th>Posterior</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(UI-25 = 0)</td>
<td>0.02</td>
<td>0.995</td>
</tr>
<tr>
<td>P(UI-10a = 0)</td>
<td>0.02</td>
<td>0.401</td>
</tr>
<tr>
<td>P(UI-10b = 0)</td>
<td>0.02</td>
<td>0.401</td>
</tr>
<tr>
<td>P(UI-10c = 0)</td>
<td>0.02</td>
<td>0.401</td>
</tr>
<tr>
<td>P(UI-10d = 0)</td>
<td>0.02</td>
<td>0.401</td>
</tr>
<tr>
<td>P(UI-10e = 0)</td>
<td>0.02</td>
<td>0.401</td>
</tr>
<tr>
<td>P(L1-2 = 0)</td>
<td>0.01</td>
<td>0.014</td>
</tr>
<tr>
<td>P(L1-1 = 0)</td>
<td>0.01</td>
<td>0.018</td>
</tr>
</tbody>
</table>

The proposed BN model also permits the computation of posterior probabilities given some known probability distribution of a state variable or set of variables. This feature expands the scope of possible studies that can be performed with the model. Table 4 shows the result LOLP of area 2 computed with different probability distributions of the assistance from the network, N2. This result shows the strong influence of the availability of network assistance on the reliability of the system.

Table 4. Computed Area 1 LOLP

<table>
<thead>
<tr>
<th>Probability</th>
<th>Distribution 1</th>
<th>Distribution 2</th>
<th>Distribution 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(N2 = 0)</td>
<td>0.0001</td>
<td>0.0005</td>
<td>0.0001</td>
</tr>
<tr>
<td>P(N2 = 15)</td>
<td>0.0328</td>
<td>0.2514</td>
<td>0.0259</td>
</tr>
<tr>
<td>P(N2 = 30)</td>
<td>0.0101</td>
<td>0.4404</td>
<td>0.1815</td>
</tr>
<tr>
<td>P(N2 = 45)</td>
<td>0.9568</td>
<td>0.3076</td>
<td>0.7924</td>
</tr>
<tr>
<td>Area LOLP, %</td>
<td>0.11</td>
<td>1.53</td>
<td>0.17</td>
</tr>
</tbody>
</table>

III. CASE STUDY

The proposed BN model has been applied in a case study of reliability analysis of a test system comprising of four areas connected through tie lines, as shown in Figure 7. Area A is the basic IEEE Reliability Test System (RTS). Generation and load data for this system can be found in [14,15]. Areas B, C and D are the three modified RTS's. the modifications are summarized in Table 5.

The RTS's annual load data in percentage provided by [13] has been used without modification. Three cases of area load dependency were considered: completely dependent, correlated and independent. In the cases of complete dependent area loads, the load curves of four areas are considered the same. In the cases of correlated area loads, the load curve in area A remains the same. The daily peak load curve (DPLC) in areas B and C are the RTS's load curve shifted forward by 4 and 2 weeks respectively. Area D load curve is the RTS's load curve shifted backward by 2 weeks.

All the tie lines have the capacity of 200 MW and FOR of 0.0005. Inter-area connection agreement is assumed as follows: i) Areas C and D are assisted by areas A and B. Area D can also assist area C by providing wheeling service; ii) Surplus capacity at sites A and B are split evenly for C and D over the tie-lines.

Figure 7. Four-area generation system

To study the effect of load uncertainty and area load dependency on the LOLP, 6 cases were considered:

- Case 1: No load uncertainty and area loads are completely dependent.
- Case 2: No load uncertainty and area loads are completely independent.
- Case 3: No load uncertainty and there is correlation between area loads.
- Cases 4, 5 and 6 are essentially the same as cases 1, 2 and 3 respectively, except that 5% standard deviation in
load forecast is introduced to consider the load uncertainty. 10% standard deviation was also examined in obtaining the LOLP results depicted in Figure 8.

A BN model for this 4-area test system has been set up using the capacity, tie-line and load models described in section II. Table 6 summarizes the calculated area LOLP indices for all the cases. The effects of load uncertainty and area load dependency on the LOLP index are depicted in Figure 8. It can be seen that load uncertainty has a strong impact on the calculated LOLP indices. LOLPs in cases with load uncertainty (cases 4 through 6) are generally higher than in the corresponding alternative cases (cases 1 through 3). Dependency of area loads also affects the LOLP. LOLP indices in the assisted areas are higher in cases when area loads are completely dependent than in the cases of partially correlated or independent area loads.

Table 6. LOLP Indices for the Cases Study

<table>
<thead>
<tr>
<th></th>
<th>Area A</th>
<th>Area B</th>
<th>Area C</th>
<th>Area D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>0.00291</td>
<td>0.005361</td>
<td>0.003383</td>
<td>0.0107</td>
</tr>
<tr>
<td>Case 2</td>
<td>0.00291</td>
<td>0.005361</td>
<td>0.002960</td>
<td>0.010441</td>
</tr>
<tr>
<td>Case 3</td>
<td>0.00291</td>
<td>0.005361</td>
<td>0.003377</td>
<td>0.01067</td>
</tr>
<tr>
<td>Case 4</td>
<td>0.00352</td>
<td>0.005927</td>
<td>0.003939</td>
<td>0.01176</td>
</tr>
<tr>
<td>Case 5</td>
<td>0.00352</td>
<td>0.005927</td>
<td>0.003681</td>
<td>0.011573</td>
</tr>
<tr>
<td>Case 6</td>
<td>0.00352</td>
<td>0.005927</td>
<td>0.004068</td>
<td>0.01178</td>
</tr>
</tbody>
</table>

The following what-if studies were performed under the assumption that maximum peak load level is observed in all the areas: a) Loss of load state in area C; b) Supply difficulty in areas A and B; c) Derated network support to area C. In situation a) the failure of the large 800MW unit in area C which has posterior probability of 0.9343 is identified as the most probable cause of the loss of load state. In situation b) the results show that the outage of generating capacity in areas A and B will likely cause significant increase in LOLP level not only in the same area but also in supported areas C and D. In situation c) the result indicates that due to the derated network support to area C, LOLP index in area C increased to 0.1137.

The studying of the test system demonstrated the robustness of the proposed BN model. On a Sun SPARC 20 work station, it takes less than 1 minute for calculating the probabilities in any of the cases study of the test system. The BN model is in nature an analytical method. However the conditional probability formulation of problems in BN models allows not only to compute the reliability indicator but also to study the effect of different factors on the value of reliability indicators. An additional advantage of BN models is the capability to studying the likelihood of the states of various system components given certain level of reliability index. The use of Monte-Carlo simulation has advantages such as: natural inclusion of real life constraints. The result of the 4-area case study provided in this paper is qualitatively comparable to the Monte-Carlo study performed for a similar system in [6] even though the numerical results can not be compared because of differences in system constraints. Both studies showed that LOLPs in cases with load uncertainty are generally higher than in the corresponding alternative cases. Both studies also showed the similar effect of dependency of area loads on the LOLP.

The test system in this case study has about 100 generating units, an average practical system size. However we considered the units grouped in 4 areas and studied only the connection between these areas. This configuration allowed limiting the size of the BN model because the connections create influences that greatly increase the size of conditional probability table in the BN model. For large highly-interconnected systems, the conditional probability table for the node "Available network support" in Figure 3 tends to become very large. This slows down computation. But the conditional probability table is sparse and recent techniques for dealing with sparse matrices in BN models could be used to address this problem [16].

IV. CONCLUSION

From this investigation, it can be concluded that Bayesian network models can be useful tools for reliability
assessment of power systems.

Proposed BN model of power system can be built conveniently by connecting together generation, transmission and load models. The model can take into consideration load uncertainty as well as dependency of load in different areas. The validation results show that the proposed model calculates the loss of load probability (LOLP) index with the same accuracy of other analytical methods. Furthermore, the BN model is uniquely capable of directly computing the posterior probabilities of variables which are most valuable for the enhanced system reliability assessment.

The results of the case studies on the 4 area test system illustrate the robustness of the proposed model. The test results indicate a considerable influence of load uncertainty on the calculated LOLP indices. Increase of load uncertainty will increase the LOLP indices. Area load dependency also exhibits similar influence pattern on the LOLP.

The results of the what-if scenario studies performed for both examples show that posterior probabilities calculated by the BN model facilitate the studying of the most likely causes and effects of observed system states. Based on this information, decision can be made for reinforcing weak points in the power network to ensure the required reliability level of a power supply system.

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V. REFERENCES


BIOGRAPHY

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