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A Bayesian network model for radiological diagnosis and procedure selection: Work-up of suspected gallbladder disease

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Bayesian networks, a technique for reasoning under uncertainty, currently are being developed for application to medical decision making. To explore their usefulness for radiologic decision support, a Bayesian belief network was constructed in the domain of hepatobiliary disease. The network model's nodes represent diagnoses, physical findings, laboratory test results, and imaging study findings. The connections between nodes incorporate conditional probabilities, such as sensitivity and specificity, to represent probabilistic influences. Statistical data were abstracted from peer-reviewed journal articles on hepatobiliary disease, and a network was created to reflect the data. The network successfully determined the *a priori* probabilities of various diseases, and incorporated laboratory and imaging results to calculate the *a posteriori* probabilities. The most informative examination was identified, that is, the laboratory study or imaging procedure that led to the greatest diagnostic certainty. Bayesian networks represent a very promising technique for decision support in radiology: they can assist physicians in formulating diagnoses and in selecting imaging procedures.

Key words: Bayesian networks, artificial intelligence, decision support systems, diagnosis, procedure selection, gallbladder disease

I. INTRODUCTION

Bayesian networks provide a formalism for reasoning about degrees of belief under conditions of uncertainty.[1] In this formalism, propositions are given numerical probability values signifying the degree of belief accorded them, and the values are combined and manipulated according to the rules of probability theory. By applying this technique to medical reasoning, one can express the relationships between diagnoses, physical findings, laboratory test results, and imaging study findings in terms of conditional probabilities such as sensitivity and specificity. Physicians can determine the *a priori* ("pretest") probability of a disease, and then incorporate laboratory and imaging results to calculate the *a posteriori* ("posttest") probability. They also can determine the most informative examination, that is, the laboratory study or imaging procedure that has a high likelihood of leading to a more certain diagnosis. We explored the usefulness of a Bayesian network model for medical decision support in the domain of hepatobiliary disease.

A Bayesian network—also called a belief network or causal probabilistic network—is a directed, acyclic graph in which nodes represent random stochastic variables, and connections between nodes represent direct probabilistic influences between the variables.[1] The graph is "directed" in that the connections between nodes have directionality, that is, they are "one way." The graph is "acyclic" in that it cannot contain cycles or "feedback" loops. Each node has two or more possible states or values; for example, the variable "gallstones" might have two possible states, "present" and "absent." Each state is associated with a probability value; for each node, these probability values sum to 1.

Typically, the direction of a connection between nodes indicates a causal influence or class–property relationship. For example, a link may indicate that gallstones influence the presence of cholecystitis. Indirect influences are represented by paths through the network. The lack of certain types of paths between two nodes indicates probabilistic independence. For example, in Fig. 1, age and sex are considered to be probabilistically independent because one does not cause the other and they do not share a common cause. The precise relation between probabilistic independence and connectedness of nodes is fairly complex and is defined in terms of a property called d-separation.[1] The strengths of influences are represented with conditional probability matrices associated with the links. For example, if node C has two parents A and B, the conditional probability matrix specifies the probabilities of the possible values that C can assume given all possible combinations of values that A and B can assume.

The primary operation performed with Bayesian networks is the computation of posterior probabilities. To do this, the values of some nodes are specified as known ("instantiated") and the probabilities of the remaining nodes, conditioned on the values of those evidence nodes, are computed by propagating the evidence through the network. Because

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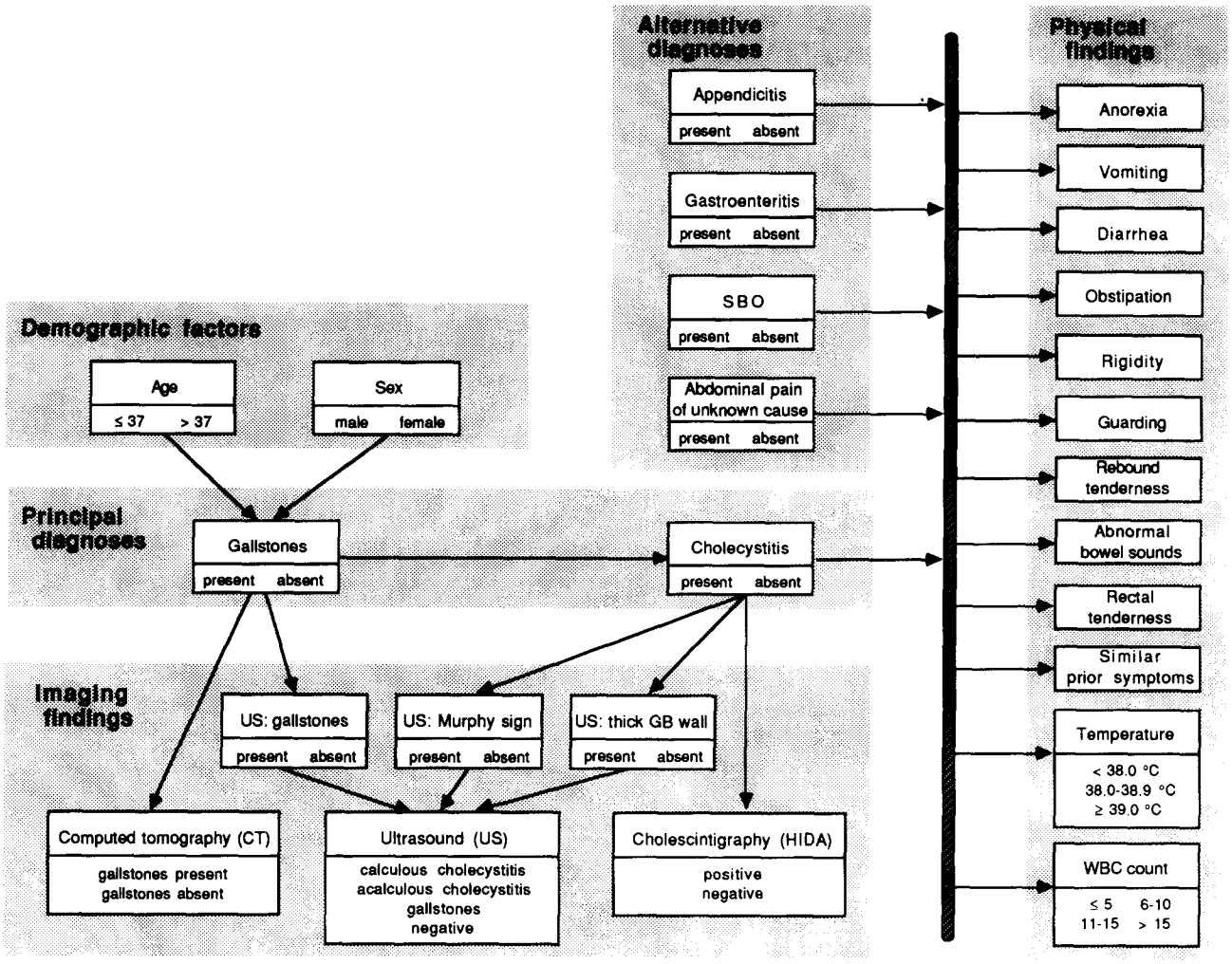

FIG. 1. Network model used for the current study. Arrows indicate the direction of influence between nodes. The vertical bar at right simplifies the illustration: all five nodes that connect to it influence all nodes to its right. The possible states of each node are shown in the lower half of each box; where not listed, the states are "present" and "absent."

links represent conditional probabilities, propagation across links is bidirectional. Several alternative algorithms exist for performing this propagation, such as that of Lauritzen and Spiegelhalter.[2] The algorithms gain efficiency by exploiting the independence information encoded in the network topology.

Bayesian networks provide a number of powerful capabilities for representing uncertain knowledge. They provide a flexible representation that allows one to specify dependence and independence of variables in a natural way through the network topology. Because dependencies are expressed qualitatively as links between nodes, one can structure the domain knowledge qualitatively before any numeric probabilities need be assigned. The graphical representation also makes explicit the structure of the domain model: a link indicates a causal relation or known association. The encoding of independencies in the network topology admits the design of efficient procedures for performing computations over the network. A further advantage of the graphical representation is the perspicuity of the resulting domain model. Finally, since Bayesian networks represent uncertainty using standard probability, one can collect the necessary data for the domain model by drawing directly on published statistical studies.

II. METHODS

A. Sources of information

Several books on radiology decision making and procedure selection[3–6] and studies published in peer-reviewed scientific journals[7–19] provided most of the data for the network's knowledge base. These sources did not contain all of the probability values necessary to construct the model. When required probability data were unavailable in the literature or sample sizes were too small, they were supplied by expert opinion or through the use of generic causal models.

One limitation of the published literature is the lack of information on the prevalence of a symptom in the context of multiple diseases. Typically, studies list only the prevalence of the symptom in the presence and absence of each disease individually. This is understandable since it is quite rare for a patient to have more than one condition at a time. If a given

symptom has several potential causes, the Bayesian network model requires conditional probabilities for all possible combinations of these causes.

B. Noisy-or model

To provide the required data, we developed a generalization of Pearl's noisy-or generic causal model.[1] Where a node is influenced by several "input" (causal) nodes, the noisy-or model augments available information by making certain assumptions about the interactions of the causes. The noisy-or model is considered superior to the "simple Bayes" and "multimembership Bayes" models for estimating joint probability distributions.[20]

Suppose we have a variable whose states can be ordered in such a way that the presence of a causal factor tends to push the variable toward higher states. Suppose further that the causes are mutually reinforcing in the sense that the more causes that are present, the more likely the variable is to be in a high state. Specifically, the more conditions that possibly cause a state e_i, the more likely the variable is to be in state e_i, and the state of the variable is the highest state caused by one of the conditions. Then if the causes are mutually independent, the influence of the multiple causes on the variable can be represented with a generalized noisy-or gate.

In our Bayesian network model of gallbladder disease, we expressed the influence of appendicitis, cholecystitis, gastroenteritis, and small bowel obstruction on each of the physical findings with a generalized noisy-or model. The model fits well for the two multistate variables, temperature and WBC count, because the presence of any one condition tends to increase the value, and one can assume that the presence of two or more conditions tends to increase the value even more.

The generalized noisy-or model is defined as follows. Suppose node E has m states, $e_1,\ldots,e_m$, which are ranked in ascending order by some metric. Node E is influenced by n condition nodes $C_1,\ldots,C_n$, where each condition C_j has only two states, "present" and "absent." Then the probability that E is in state e_i is just the probability that at least one of the C_j causes e_i and no C_j causes a higher state. Given the independence assumption, this is

$$P(E=e_i|\text{conditions } C_j) = P(\text{all } C_j \text{ cause a state} \leq i) - P(\text{all } C_j \text{ cause a state} < i). \quad (1)$$

In Pearl's noisy-or model, the effect variable E is permitted to have only two states; this is a special case of our generalized noisy-or model. The formal derivation of the generalized noisy-or model is set out in the Appendix.

C. Gallbladder disease model

Our belief network model is shown in Fig. 1. We assume that the patient presents with acute abdominal pain. The network contains two diagnoses of interest: gallstones and acute cholecystitis. Four diagnoses serve as alternative causes of acute abdominal pain: appendicitis, gastroenteritis, small bowel obstruction (SBO), and abdominal pain of unknown cause. Each diagnosis node has two states, "present" and "absent;" each state has an associated probability value. In essence, then, these nodes represent the probabilities that the disorders are present. The presence of gallstones influences the probability that cholecystitis is present. Age and sex are background factors that influence the presence of gallstones. Because the nodes for the alternative diagnoses and demographic variables are not influenced by any "parent" nodes, their *a priori* (pretest) probability values must be defined explicitly (Table I).

TABLE I. Assumptions for Bayesian network model. For nodes where only the "present" state is indicated, the *a priori* probability of the "absent" state is $1-P$, where P is the probability of the "present" state.

Node	Value	Probability	Source
Age	≤37	0.40	(estimate)
	>37	0.60	
Sex	Male	0.40	(estimate)
	Female	0.60	
Appendicitis	present	0.0754	9
Gastroenteritis	present	0.1193	9
Small bowel obstruction (SBO)	present	0.0474	9
Abdominal pain of unknown cause	present	0.7123	9

The remaining nodes represent the patient's history, symptoms, signs, and test results. Anorexia, vomiting, diarrhea, obstipation, rigidity, guarding, rebound tenderness, abnormal bowel sounds, rectal tenderness, and "similar prior symptoms" are symptoms and signs of cholecystitis and the four alternative diagnoses. These nodes have two possible states, present and absent. Two other nodes are related to these five disorders: temperature and white blood cell (WBC) count. Temperature has three possible states: "normal" (<38.0 °C), "low fever" (38.0–38.9 °C), and "high fever" (≥39.0 °C). The WBC count node has four states (expressed in 10^3 WBCs/mm^3): 0–5, 6–10, 11–15, and greater than 15.

The model contains two imaging tests for gallstones, ultrasound (US) and computed tomography (CT); the links to these tests from the diagnoses include values of sensitivity and specificity. We considered three imaging findings for cholecystitis: the sonographic Murphy sign (maximal tenderness upon gallbladder compression during ultrasound examination), thickened gallbladder wall by ultrasound, and radionuclide hepatobiliary imaging ("HIDA"). Conditional probabilities for the connections to these nodes are listed in Table II. The nodes for these tests have two states: positive and negative. The node labeled "ultrasound" consolidates the results of the three different ultrasound tests; the link matrix relating it to the three tests contains only zeros and ones.

Even though we were not interested in diagnosing appendicitis, gastroenteritis, or SBO, it was necessary to include them in order to properly model the domain. These conditions are included in the model so that observed symptoms do not inordinately increase the probability of cholecystitis. If cholecystitis were the only represented cause of a symptom such as vomiting, then the presence of vomiting would produce a high probability of cholecystitis. The inclusion of

TABLE II. Conditional probabilities used in Bayesian network model: CT =computed tomography, US=ultrasound, HIDA=hepatobiliary scintigraphy.

Node (Value)	Input conditions		Probability	Source
Gallstones				
	Age	Sex		
present	≤37	male	0.05	(estimates)
		female	0.07	
	>37	male	0.13	
		female	0.20	
Cholecystitis				
	Gallstones			
present	present		0.197	9,18
	absent		0.00435	
US gallstones				
	Gallstones			
present	present		0.955	18
	absent		0.186	
CT gallstones				
	Gallstones			
present	present		0.790	7
	absent		0.003	(estimate)
US Murphy sign				
	Cholecystitis			
present	present		0.719	18
	absent		0.124	
US thick gallbladder wall				
	Cholecystitis			
present	present		0.453	18
	absent		0.116	
HIDA				
	Cholecystitis			
positive	present		0.923	17
	absent		0.161	

other possible causes of vomiting attenuates the degree to which the symptom provides evidence for any single cause by providing alternative explanations of the symptom.

D. Analysis

The analyses were carried out using the HUGIN inference system.[21] The system allows one to fix ("instantiate") the values of any nodes in the network and to infer the probabilities of the remaining nodes conditioned on these values. There are 12 "physical findings" variables that represent symptoms and history, 10 of which have two states (present/absent); temperature has three states and WBC count has four states. Hence, there are $2^{10} \cdot 3 \cdot 4 = 12\,288$ possible combinations of presenting symptoms. By combining these with the four different combinations of age and sex, there are 49 152 possible presentations. We determined the effectiveness of each test to rule in and rule out both gallstones and acute cholecystitis by positively and negatively instantiating each test outcome for various configurations of age, sex, and physical findings.

The information obtained from a test result is the absolute value of the difference between the pretest and posttest probabilities of disease. To assess the value of each imaging modality, we defined two measures, informativeness and expected informativeness. The informativeness, I, of a test for a disease is the sum of the information obtained about the disease for both positive and negative test outcomes. The expected informativeness, $E(I)$, is the sum of the information obtained about the disease for both positive and negative test outcomes, with each term weighted by the likelihood of observing that test outcome. That is,

$$I = |P(D|s,T) - P(D|s)| + |P(D|s,\neg T) - (D|s)|, \quad (2)$$

$$E(I) = P(T|s) \cdot |P(D|s,T) - P(D|s)| + P(\neg T|s) \cdot |P(D|s,\neg T) - P(D|s)|, \quad (3)$$

where D indicates the presence of the disease, T means the test is positive, $\neg T$ indicates the test is negative, and s is the constellation of symptoms. We denote the pretest probability of disease, given the known symptoms, as $P(D|s)$, and the posttest probability, given the known symptoms and a positive test result, as $P(D|s,T)$. We compared CT and ultrasound for the diagnosis of gallstones, and hepatobiliary scintigraphy (HIDA) and ultrasound for the diagnosis of cholecystitis.

III. RESULTS

A. Gallstones

Figure 2 displays the effectiveness of CT and ultrasound for diagnosis of gallstones. Figure 2(a) shows the posttest vs pretest probability for both tests. Each pretest probability was computed by instantiating a possible combination of values for the demographic factors and physical findings and propagating this evidence to determine its influence on gallstones. The posttest probability was then computed by additionally instantiating the outcome of the imaging finding and propagating the information through the network. To determine the effectiveness of CT, we instantiated the node labeled "Computed tomography" and to determine the effectiveness of ultrasound we instantiated the node labeled "US gallstones." For example, at a pretest probability of gallstones of .18, a positive ultrasound test produces a posttest probability of .53 and a negative ultrasound test produces a posttest probability of .01. Because of its high specificity, CT is the optimal modality to "rule in" gallstones at any pretest probability of gallstones; its positive posttest probability approximates unity. Likewise, ultrasound is the preferred test to rule out gallstones. Unlike a receiver operating characteristic (ROC) curve, Fig. 2(a) shows which procedure yields a more certain diagnosis for each combination of history and symptoms for both positive and negative test outcomes.

The informativeness of CT for gallstones is greater than that of ultrasound when the pretest probability of gallstones is low [Fig. 2(b)]. If the pretest probability of gallstones exceeds about .55, then ultrasound, because of its ability to rule

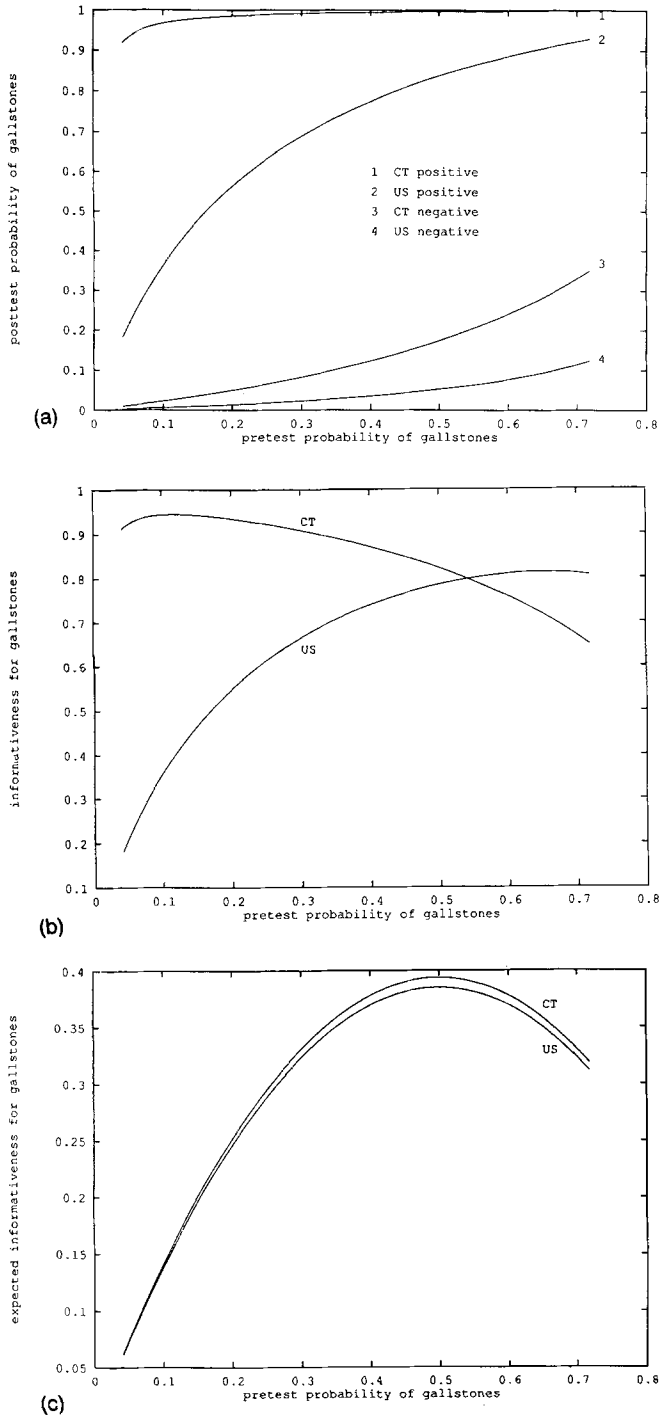

FIG. 2. Comparison of computed tomography (CT) and ultrasound (US) examinations for detection of gallstones. (a) Pretest vs posttest probabilities for positive and negative findings of gallstones on CT and US. (b) Informativeness of CT and US for diagnosis of gallstones. (c) Expected informativeness of CT and US for diagnosis of gallstones.

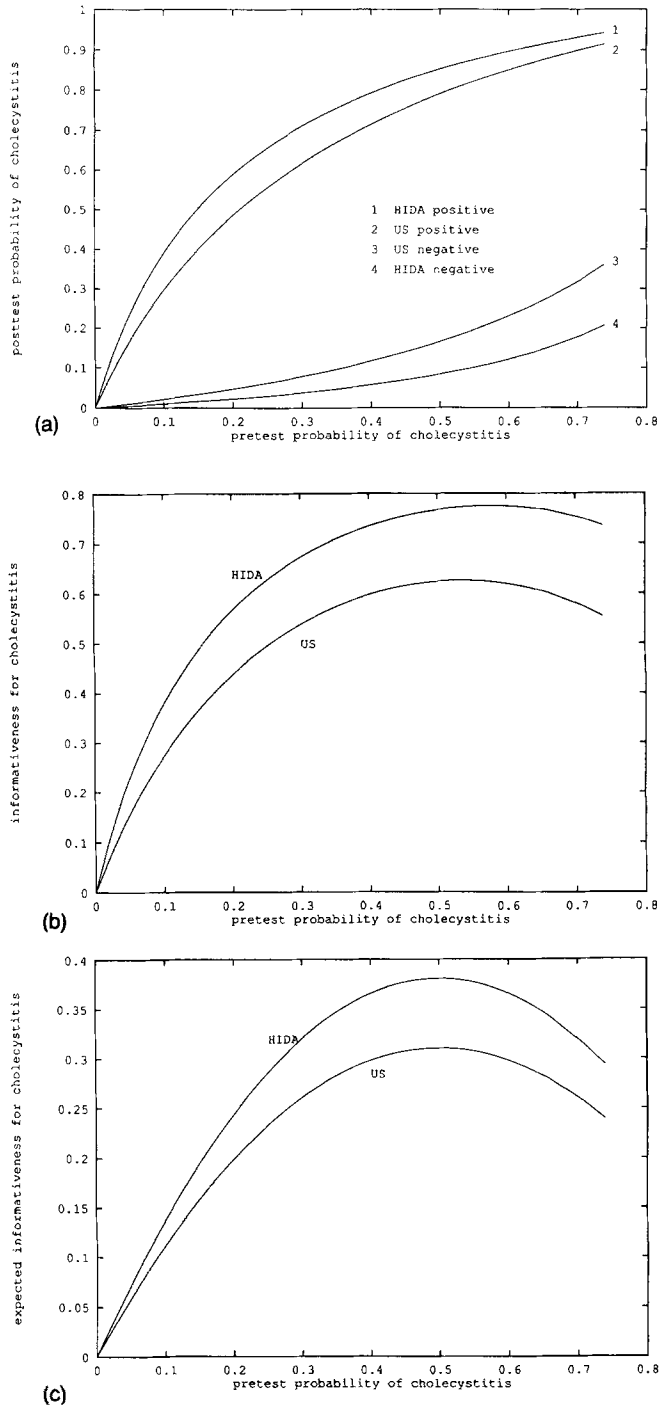

FIG. 3. Comparison of hepatobiliary scintigraphy (HIDA) and ultrasound (US) examinations for detection of cholecystitis. (a) Pretest vs posttest probabilities for positive and negative findings of cholecystitis on HIDA and US. (b) Informativeness of HIDA and US for diagnosis of cholecystitis. (c) Expected informativeness of HIDA and US for diagnosis of cholecystitis.

out disease, becomes more informative. However, the expected informativeness of CT exceeds that of ultrasound at any pretest probability of gallstones [Fig. 2(c)]. Note that both tests have the greatest expected informativeness at a prior probability of .5, where "uncertainty" is greatest.

B. Cholecystitis

The performance of ultrasound and hepatobiliary scintigraphy ("HIDA") are compared in Figure 3. Figure 3(a) displays the posttest vs pretest probability of cholecystitis. To determine the posttest probability of HIDA, we positively and negatively instantiated the node labeled "Cholescintigraphy." To define positive and negative outcomes for ultrasound, we introduced a dummy node that represented the logical disjunction of the nodes "US: Murphy sign" and "US: Thick GB wall." Ultrasound thus was considered positive if either the sonographic Murphy sign or gallbladder wall thickening was present; the posttest probability of ultra-

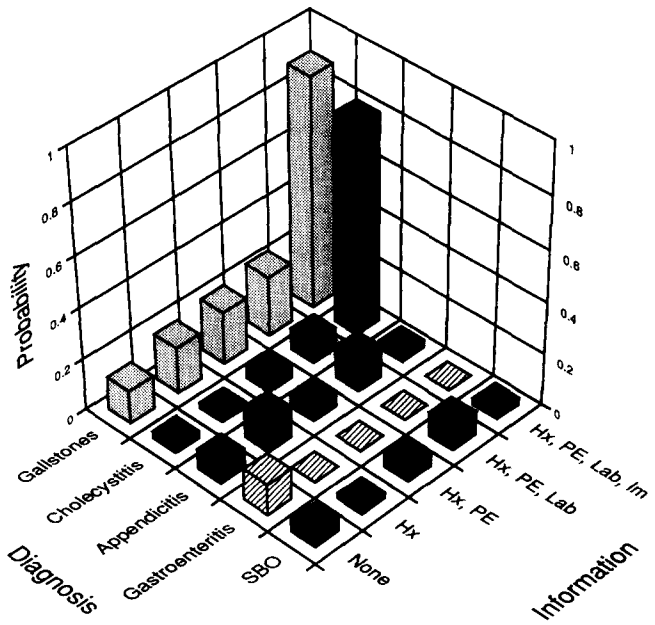

FIG. 4. Diagnostic probabilities based on information available in each stage of the work-up of a hypothetical patient with acute abdominal pain. Information includes patient's history (Hx), physical examination (PE), laboratory tests (Lab), and imaging procedures (Im). See text.

sound was then determined by positively and negatively instantiating this dummy node. HIDA demonstrates superior positive and negative predictive values, that is, it offers greater certainty for both positive and negative test outcomes [Fig. 3(a)]. HIDA also yields the greatest informativeness and expected informativeness over the entire range of pretest probabilities of cholecystitis [Figs. 3(b) and 3(c)].

C. Case simulation

To illustrate the use of the belief network model, we presented information about a specific patient to the system in sequential segments, as is common in clinical practice and in clinical reasoning exercises. Figure 4 shows the probabilities of all diagnoses except "abdominal pain of unknown cause." Initially, no information ("None") is available about the specific patient: the probability values indicate the *a priori* probability (prevalence) of each disorder for patients with acute abdominal pain. The cumulative information provided at each stage of the patient's work-up, specified below, alters the probabilities that the diagnoses are present.

History (Hx). "A 41-year-old woman presents with anorexia and acute abdominal pain; she denies vomiting, diarrhea, obstipation or similar previous symptoms." Gallstones and appendicitis become slightly more likely. The probability of gastroenteritis approximates zero.

Physical examination (PE). "Guarding is present. There is no rigidity or rebound tenderness. Bowel sounds are probably normal ($P=.80$). Rectal examination was deferred; thus, rectal tenderness was not evaluated. The patient's temperature is 38.6 °C." Gallstones and cholecystitis become slightly more probable.

Laboratory tests (Lab). "The patient's WBC count is 12,600 cells/mm^3." The probability of appendicitis increases to .12. The probabilities of gallstones, cholecystitis, and small bowel obstruction increase slightly. Small bowel obstruction ($P=.09$) remains slightly more likely than cholecystitis ($P=.08$), and gastroenteritis remains a virtual impossibility.

Imaging procedures (Im). "Ultrasound examination reveals gallstones and probable gallbladder wall thickening [$P=.70$]. The sonographic Murphy sign is present." Now gallstones and cholecystitis become the most likely diagnoses, with probabilities of .89 and .81, respectively. Based on the strong evidence for cholecystitis, the probabilities of appendicitis and SBO fall to about .03, which essentially excludes these diagnoses from consideration.

IV. DISCUSSION

Several techniques from the discipline of artificial intelligence have been applied to the problems of radiological diagnosis and procedure selection, including rule-based systems,[22,23] artificial neural networks,[24–26] and hypertext systems.[27,28] Bayesian networks represent a relatively new formalism, and recently have been applied to several medical domains.[29–32]

Bayesian networks can successfully model the information needed to determine diagnoses and select imaging procedures. The network provides a simple, concise method of representing the interactions among the various components of the domain model. Bayesian networks can explain their reasoning by referring directly to the topology of the network and the conditional probabilities that form the connections between nodes; in contrast, artificial neural networks cannot explain their reasoning. Bayesian networks employ conditional probabilities obtained from studies of patient populations, a more accurate and quantitative approach than the "certainty factors" employed in most rule-based expert systems.

For a specific patient presentation, a Bayesian network can calculate the probabilities of the presence of several disorders so that a physician can formulate a rational differential diagnosis. The techniques also can be used to assess the utility of various diagnostic imaging procedures over the entire spectrum of physical and laboratory findings. One can assess the utility of imaging procedures in different patient populations by altering the *a priori* (pretest) probability values of factors such as age and sex. Patient-specific data can include the lack of information ("rectal tenderness not evaluated") and uncertain observations ("bowel sounds probably normal"). If the results of one or more imaging procedures are available, these too can be applied in the model; in this manner, one can determine the added certainty (if any) that additional imaging procedures would offer. Such a system might help reduce physicians' overutilization of imaging procedures in their quest for diagnostic certainty.[33]

How sensitive is our model to changes in the conditional probability values? The topology of a Bayesian network—which allows for conditional independence of variables—localizes the effects of changes in conditional probability values. In our network model, once the pretest probability of a disease is established from the values of age, sex, and physical findings, the posttest probability is a function only

of the sensitivity and specificity of each imaging finding. Sensitivity analysis of probability values in Bayesian networks is a topic of active research.[20,34]

We are developing software to allow physicians and medical students to examine the probabilities of diseases for specific combinations of symptoms and test results. This software will formulate explanations of the system's reasoning to be used for both clinical consultation and computer-aided instruction. We will use this software to present hypothetical case scenaria to physicians to validate our model and to assess physicians' judgment about probabilities of disease.

Bayesian networks represent a very promising technique for clinical decision support in radiology. They can assist physicians in determining the most likely diagnoses and in selecting imaging procedures. One drawback is the need for a large amount of reliable, quantitative data to construct the network model. Further effort is necessary to develop more comprehensive models of interactions between demographic factors, symptoms and diseases; effort is underway to convert the knowledge base of a large, rule-based expert system for internal medicine (QMR) into a Bayesian network model.[30,31]

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APPENDIX: GENERALIZED NOISY-OR MODEL

The generalized noisy-or model is defined as follows. Suppose node E has m states, $e_1,\ldots,e_m$, which are ranked in ascending order by some metric. Node E is influenced by n condition nodes $C_1,\ldots,C_n$, where each condition C_j has only two states, "present" and "absent." Define $q_{ij} = P(E=e_i|C_j \text{ present})$, that is, the probability that node E attains state e_i given that condition C_j is present. Given these values, we need to determine the link matrix that expresses the conditional probabilities of E for all possible combinations of states of $C_1,\ldots,C_n$.

Let T be the subset of $\{C_1,\ldots,C_n\}$ that contains all conditions C_j that are present. Then, since the causes are mutually reinforcing for state e_m, the "highest" state, we have

$$P(E=e_m|T) = P(\text{at least one condition } C_j \text{ causes } e_m) \tag{A1}$$

$$= 1 - \prod_{C_j \in T} (1-q_{mj}) \tag{A2}$$

$$= 1 - \prod_{C_j \in T} (q_{1j}+q_{2j}+\cdots+q_{(m-1)j}). \tag{A3}$$

Note that $(1-q_{mj})$ is the probability that C_j does not cause e_m. For state e_i, where $1 \leq i < m$,

$$P(E=e_i|T) = P(\text{at least one } C_j \text{ causes } e_i \text{ and no } C_j \text{ causes a higher state}) \tag{A4}$$

$$= (1 - P(\text{no } C_j \text{ causes } e_i | \underset{k>i}{\Lambda} \text{ no } C_j \text{ causes } e_k)) \cdot P(\text{no } C_j \text{ causes } e_{i+1} | \underset{k>i+1}{\Lambda} \text{ no } C_j \text{ causes } e_k)$$
$$\cdot P(\text{no } C_j \text{ causes } e_{i+2} | \underset{k>i+2}{\Lambda} \text{ no } C_j \text{ causes } e_k) \cdot \cdots \cdot P(\text{no } C_j \text{ causes } e_m) \tag{A5}$$

$$= \left(1 - \frac{P\left(\underset{k\geq i}{\Lambda} \text{ no } C_j \text{ causes } e_k\right)}{P\left(\underset{k>i}{\Lambda} \text{ no } C_j \text{ causes } e_k\right)}\right) \cdot \frac{P\left(\underset{k\geq i+1}{\Lambda} \text{ no } C_j \text{ causes } e_k\right)}{P\left(\underset{k>i+1}{\Lambda} \text{ no } C_j \text{ causes } e_k\right)}$$
$$\cdot \frac{P\left(\underset{k\geq i+2}{\Lambda} \text{ no } C_j \text{ causes } e_k\right)}{P\left(\underset{k>i+2}{\Lambda} \text{ no } C_j \text{ causes } e_k\right)} \cdots \cdot P(\text{no } C_j \text{ causes } e_m), \tag{A6}$$

where Λx_i denotes the logical conjunction of x_i, that is, the condition x_1 AND x_2 AND $\cdots$ AND x_n. Thus, by the independence assumption,

$$P(E=e_i|T) = \left(1 - \frac{\prod_{C_j \in T} P\left(\underset{k\geq i}{\Lambda} C_j \text{ does not cause } e_k\right)}{\prod_{C_j \in T} P\left(\underset{k>i}{\Lambda} C_j \text{ does not cause } e_k\right)}\right) \cdot \frac{\prod_{C_j \in T} P\left(\underset{k\geq i+1}{\Lambda} C_j \text{ does not cause } e_k\right)}{\prod_{C_j \in T} P\left(\underset{k>i+1}{\Lambda} C_j \text{ does not cause } e_k\right)}$$
$$\cdot \frac{\prod_{C_j \in T} P\left(\underset{k\geq i+2}{\Lambda} C_j \text{ does not cause } e_k\right)}{\prod_{C_j \in T} P\left(\underset{k>i+2}{\Lambda} C_j \text{ does not cause } e_k\right)} \cdots \cdot \prod_{C_j \in T} P(C_j \text{ does not cause } e_m) \tag{A7}$$

$$= \left[1 - \prod_{C_j \in T} \left(\sum_{k \leq i-1} q_{kj} \Big/ \sum_{k \leq i} q_{kj} \right)\right] \left[\left(\prod_{C_j \in T} \left(\sum_{k \leq i} q_{kj} \Big/ \sum_{k \leq i+1} q_{kj} \right)\right)\right]$$

$$\times \left[\left(\prod_{C_j \in T} \left(\sum_{k \leq i+1} q_{kj} \Big/ \sum_{k \leq i+2} q_{kj} \right)\right)\right] \cdots \left(\prod_{C_j \in T} (1 - q_{mj}) \right) \tag{A8}$$

$$= \prod_{C_j \in T} (q_{1j} + q_{2j} + \cdots + q_{ij}) - \prod_{C_j \in T} (q_{1j} + q_{2j} + \cdots + q_{(i-1)j}). \tag{A9}$$

Note that this equation also holds for $i=m$, since $(q_{1j}+q_{2j}+\cdots+q_{mj})=1$. Our model can be shown to be a special case of Srinivas' generalization of the noisy-or model,[35] which accomodates multistate variables and discrete functions other than the Boolean OR function.

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